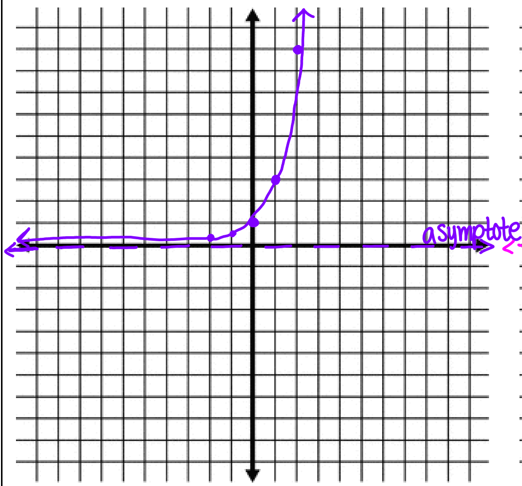
Graphing Exponential Functions

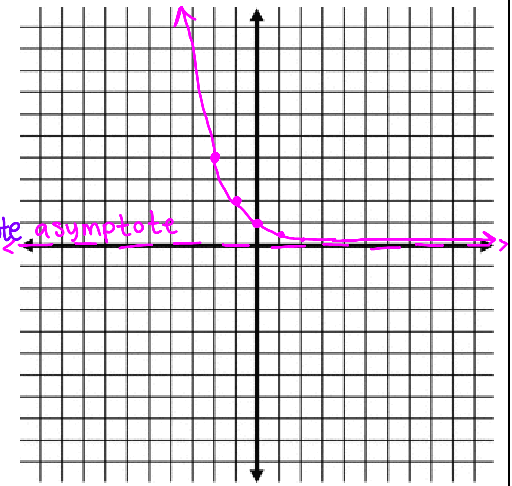
Example: $f(x)=3^x$

x	$f(x)=3^x$	$(x, f(x))$
-2	$3^{-2}=\frac{1}{9}\approx 0.1$	(-2, 0.1)
-1	$3^{-1}=\frac{1}{3}\approx 0.3$	(-1, 0.3)
0	$3^0=1$	(0, 1)
1	$3^1=3$	(1, 3)
2	$3^2=9$	(2, 9)



Example: $g(x)=(\frac{1}{2})^x$

x	$g(x)=(\frac{1}{2})^x$	$(x, f(x))$
-2	$(\frac{1}{2})^{-2}=4$	(-2, 4)
-1	$(\frac{1}{2})^{-1}=2$	(-1, 2)
0	$(\frac{1}{2})^0=1$	(0, 1)
1	$(\frac{1}{2})^1=\frac{1}{2}=0.5$	$(1, \frac{1}{2})$
2	$(\frac{1}{2})^2=\frac{1}{4}=0.25$	$(2, \frac{1}{4})$



Writing Exponential Functions

Example: $f(x)$ increases from $(-\infty,\infty)$ and contains the point (2,9).

growth

$f(x)=b^x$

$9=b^2$

$b=3$

$\boxed{f(x)=3^x}$

Example: $g(x)$ decreases from $(-\infty,\infty)$ and contains the point (-2,25).

decay

$g(x)=b^x$

$25=b^{-2}$ any negative exponents flip fractions

$b=\frac{1}{5}$

$\boxed{g(x)=(\frac{1}{5})^x}$

Transformations of Exponential Functions

$f(x)$ is the original function
$g(x)$ is the new function

$g(x)=Af(Bx-C)+D$ **OR** $g(x)=Ab^{(Bx-C)}+D$

C: shift left or right

B: horizontal reflection (-x)

A: vertical stretch or compression AND vertical reflection

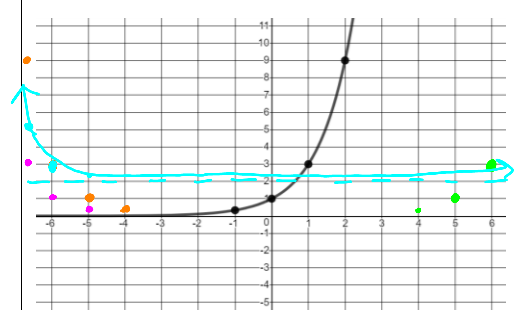
D: shift up or down

Example: Write out how $f(x)=3^x$ has been transformed to get $g(x)$. Then, graph $g(x)$ on the same graph as $f(x)$.

$$g(x)=\frac{1}{3}(3)^{(-x-5)}+2$$

$f(x)$ was shifted right 5 units, horizontally reflected, vertically compressed by $\frac{1}{3}$, and shifted up 2 units

to get $g(x)$.



Exponential Function Word Problem #1

Formula: $A(t)=P(1+r)^t$

P: initial amount (original/starting)

r: rate (NOT as a %)

t: time

A medication decays at a rate of 15% per hour in the body. If the initial amount given is 500 mg, how much will be left in the body after 12 hours?

-0.15

$A(12)=500(1-0.15)^{12}$

$A(12)=500(0.85)^{12}$

$A(12)=71.12$

There will be 71.12 mg left after 12 hours.

Exponential Function Word Problem #2

Formula: $A(t)=P\left(1+\frac{r}{n}\right)^{nt}$

P: initial amount (original/starting)

r: rate (NOT as a %)

n: number of times compounded

Quarterly: 4
Monthly: 12
Weekly: 52
Daily: 365

t: time in years

Karla put $100.00 into a bank account for her future car. The account earns 2% compounded weekly. How much will be in the account after 15 years?

$A(15)=100\left(1+\frac{0.02}{52}\right)^{52\cdot 15}$

$A(15)=134.978$

There will be $134.98 after 15 years.

Exponential Function Word Problem #3

Formula: $N(t)=N_0e^{rt}$

N_0: initial amount (original starting)

r: rate (NOT as a %)

t: time in years

The population of bees has been decreasing by 1.2% each year. If there are 120,000 bees in Anaheim in 2019, how many will there be in 2024? How many were there in 2001?

$N(5)=120{,}000e^{-0.012\cdot 5}$

$N(5)=120{,}000e^{-0.06}$

$N(5)=113{,}011.74$

There will be 113,012 bees in 2024.

$N(-18)=120{,}000e^{-0.012\cdot(-18)}$

$N(-18)=120{,}000e^{0.216}$

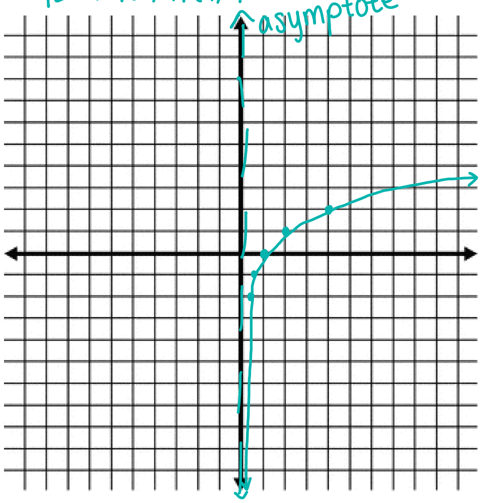
$N(-18)=148{,}932.29$

There were 148,932 bees in 2001.

Graphing Logarithmic Functions

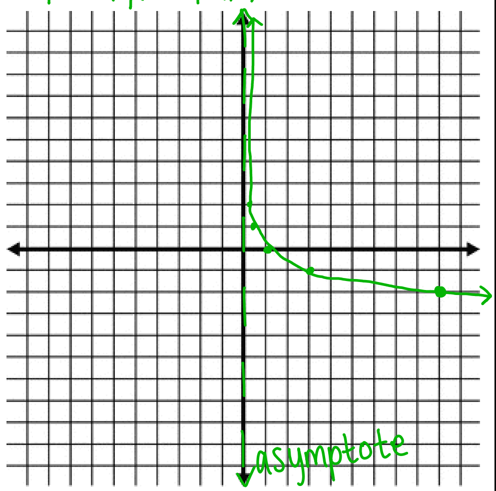
Example: $f(x)=\log_2 x$

x	$y=2^x$	(x,y)	(y,x)
-2	$2^{-2}=\frac{1}{4}$	$(-2,\frac{1}{4})$	$(\frac{1}{4},-2)$
-1	$2^{-1}=\frac{1}{2}$	$(-1,\frac{1}{2})$	$(\frac{1}{2},-1)$
0	$2^0=1$	$(0,1)$	$(1,0)$
1	$2^1=2$	$(1,2)$	$(2,1)$
2	$2^2=4$	$(2,4)$	$(4,2)$



Example: $f(x)=\log_{1/3} x$

x	$y=(\frac{1}{3})^x$	(x,y)	(y,x)
-2	$(\frac{1}{3})^{-2}=9$	$(-2,9)$	$(9,-2)$
-1	$(\frac{1}{3})^{-1}=3$	$(-1,3)$	$(3,-1)$
0	$(\frac{1}{3})^0=1$	$(0,1)$	$(1,0)$
1	$(\frac{1}{3})^1=\frac{1}{3}$	$(1,\frac{1}{3})$	$(\frac{1}{3},1)$
2	$(\frac{1}{3})^2=\frac{1}{9}$	$(2,\frac{1}{9})$	$(\frac{1}{9},2)$



Rewriting Logarithms & Exponentials

$y=b^x \rightarrow$ $\log_b y = x$

Example: Rewrite in logarithmic form.

1. $2^3 = 8$

   $\log_2 8 = 3$

2. $(\frac{1}{2})^y = 4$

   $\log_{\frac{1}{2}} 4 = y$

Example: Solve for x.

1. $\log_4 x=2$

   $4^2 = x$ $\boxed{x=16}$

2. $\log_5 125=x$

   $5^x = 125$

   $5^3 = 125$ $\boxed{x=3}$

Transformations of Logarithmic Functions

f(x) is the original function

g(x) is the new function

$g(x)=Af(Bx-C)+D$ **<u>OR</u>** $g(x)=A\log_b(Bx-C)+D$

C: __________

B: __________

A: __________

____

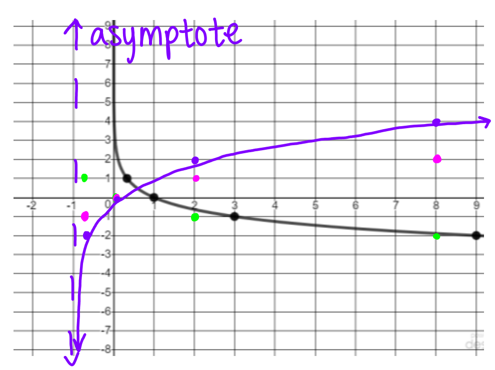
D: __________

Example: Write out how $f(x)=\log_{1/3} x$ has been transformed to get g(x). Then, graph g(x) on the same graph as f(x).

$$g(x) = -2\log_{\frac{1}{3}}(x+1)$$

f(x) was shifted 1 unit left, vertically reflected, and vertically stretched by 2

to get g(x).



Special Logarithms

$\log x \rightarrow$ $\log_{10} x$

$\ln x \rightarrow$ $\log_e x$

Example: Solve for x.

1. $\log 1000=x$

   $\log_{10} 1000 = x$

   $10^x = 1000$ $\boxed{x=3}$

   $10^3 = 1000$

2. $\ln y=7$

   $\log_e y = 7$

   $e^7 = y$ $\boxed{y=1096.63}$

3. $\log x=100$

   $\log_{10} x = 100$

   $\boxed{10^{100} = x}$

Characteristics of Functions

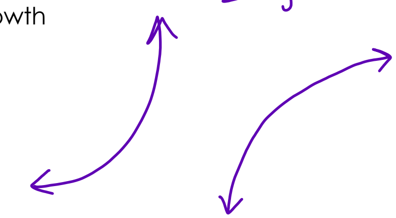
DOMAIN: (smallest x, largest x) L R

RANGE: (smallest y, largest y) T B

Asymptote: the line the graph almost touches

x= OR y=

Growth



Decay