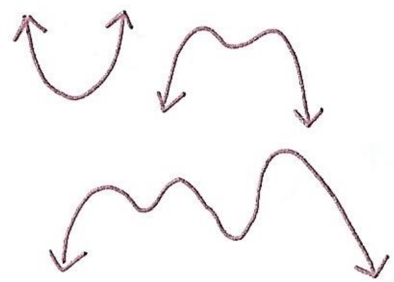
Even Power Functions

Example: $x^2, -x^4$

Possible Shapes:



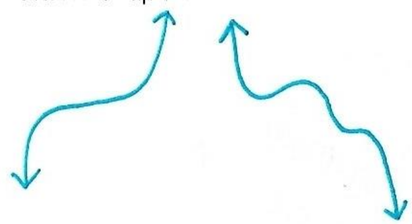
Possible End Behaviors:

1. As $x \to \infty$, $y \to$ ∞.
   As $x \to -\infty$, $y \to$ ∞. +a
2. As $x \to \infty$, $y \to$ $-\infty$.
   As $x \to -\infty$, $y \to$ $-\infty$. −a

Odd Power Functions

Example: $x^3, -x^5$

Possible Shapes:



Possible End Behaviors:

1. As $x \to \infty$, $y \to$ ∞.
   As $x \to -\infty$, $y \to$ $-\infty$. +a
2. As $x \to \infty$, $y \to$ $-\infty$.
   As $x \to -\infty$, $y \to$ ∞. −a

Transformations of Power Functions

f(x) is the original function
g(x) is the new function

$g(x)=Af(B(x-C))+D$

C: shifted left (+) or right (−)

B: reflected horizontally
hor. compress ($B>1$) / hor. stretch ($B<1$)

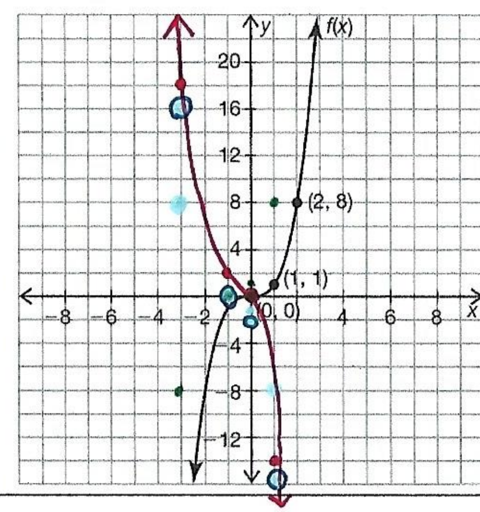
A: vertically reflected
ver. compress ($A<1$) / ver. stretch ($A>1$)

D: shifted up (+) / down

Example: Write out how f(x) has been transformed to get g(x). Then, graph g(x) on the same graph as f(x).

$$g(x) = -2f(x+1)+2$$

f(x) was shifted 1 unit left, vertically reflected, vertically stretched by 2, and shifted 2 units up to get g(x).



Transformations of Power Functions

Example: Write the transformational function notation given the description.

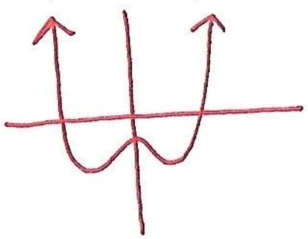
f(x) has been horizontally shifted 7 units right, reflected horizontally, vertically compressed by ½, and vertically shifted down 2 units.

$$g(x) = \frac{1}{2} f(-x-7)-2$$

Even Symmetry

A graph that is symmetric about the y-axis.

Graphical Example:



Verify whether the function is even.

$f(x)=2x^4-3x^2+6$

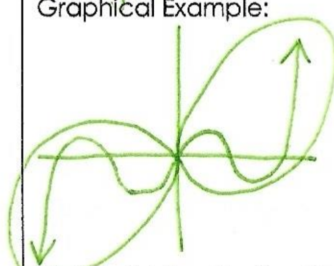
$f(-x)=2(-x)^4-3(-x)^2+6$
$=2(x^4)-3(x^2)+6$
$=2x^4-3x^2+6$

$\boxed{f(x)=f(-x)}$ even ✓

Odd Symmetry

A graph that is symmetric about the origin.

Graphical Example:



Diagonals

Verify whether the function is even.

$f(x)=5x^3-7x$

$f(-x)=5(-x)^3-7(-x)$
$=5(-x^3)+7x$
$=-5x^3+7x$

$-f(x)=-5x^3-7x$

$-f(x)=f(-x)$ odd ✓

X-Intercepts

Fundamental Theorem of Algebra: "Any polynomial function of degree n must have exactly n complex or real roots."

If a 4th degree function has 2 real x-intercepts, how many complex x-intercepts would it have? 2

How many real x-intercepts could a 5th degree function have?
5, 4, 3, 2, or 1

How many real x-intercepts could an 8th degree function have?
8, 7, 6, 5, 4, 3, 2, 1, or 0

Extrema

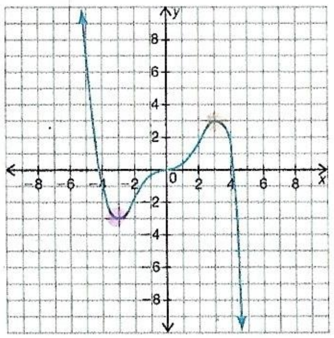
Absolute Maximum:
tallest in entire graph

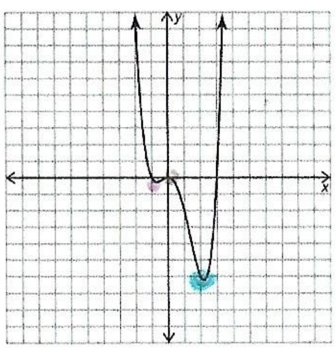
Absolute Minimum:
shortest in entire graph

Relative Maximum:
tallest in section of graph

Relative Minimum:
shortest in section of graph

Label the extrema that this function has.





Does an odd degree function ever have absolute extrema? Yes/No

How many extrema could a 7th degree function have?
6, 4, 2, 0

How many extrema could a 6th degree function have?
5, 3, 1

Multiplicity

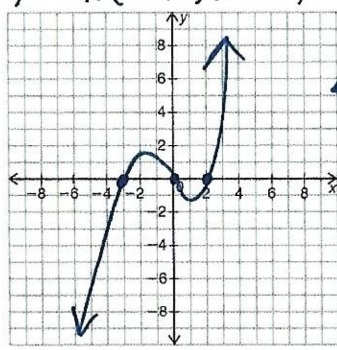
The number of times a number is a root for a given polynomial function.

Write 2 different functions with the given characteristics and graph one of them.

1. X-Intercepts: x=2, x= - 3, and x=0

$f(x) = (x-2)(x+3)x$

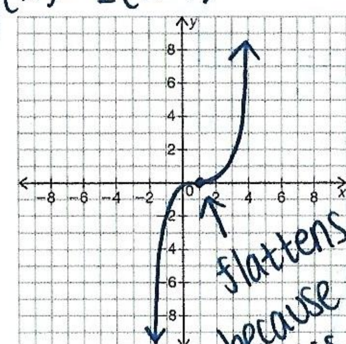
$g(x) = -x(x-2)(x+3)$



2. X-Intercepts: x=1 (Multiplicity 3)

$f(x) = (x-1)^3$

$g(x) = 2(x-1)^3$



flattens because mult is bigger

Does an x-intercept with even multiplicity bounce or cross? bounce

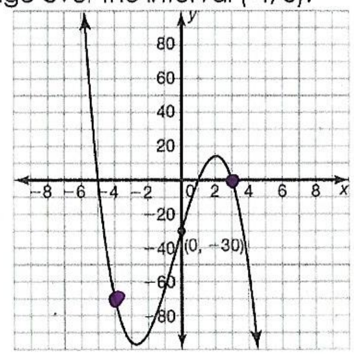
Does an x-intercept with odd multiplicity bounce or cross? cross

Average Rate of Change

Formula:

$$\frac{f(b)-f(a)}{b-a}$$

Example: Find the average rate of change over the interval (-4, 3).



$$\frac{f(3)-f(-4)}{3-(-4)} = \frac{0-(-70)}{3+4}$$

$$= \frac{0+70}{7}$$

$$= \frac{70}{7} = \boxed{10}$$

Polynomial Long Division

Example: Write the dividend as the product of the quotient and the divisor.

$(2x^3+3x^2+7x+5)\div(2x+1)$

$$x^2+x+3$$

$$2x+1 \overline{)\,\cancel{2x^3}+3x^2+7x+5}$$

$$-2x^3-x^2$$

$$2x^2+7x+5$$

$$-2x^2-x$$

$$6x+5$$

$$-6x-3$$

$$2$$

$$(2x^3+3x^2+7x+5) = \left(x^2+x+3+\frac{2}{2x+1}\right)(2x+1)$$

Synthetic Division

Example: Write the dividend as the product of the quotient and the divisor.

$(x^3-9x^2+8x+60)\div(x-5)$

5	1	-9	8	60
		5	-20	-60
	1	-4	-12	0

$$x^3-9x^2+8x+60 = (x-5)(x^2-4x-12)$$

$(4x^3-5x-12)\div(2x+1)$

$-\frac{1}{2}$	4	0	-5	-12
		-2	1	2
	$\frac{4}{2}$	$\frac{-2}{2}$	$\frac{-4}{2}$	-10

$$4x^3-5x-12 = (2x+1)\left(2x^2-x-2-\frac{10}{2x+1}\right)$$

Factoring

$a^2-b^2 =$ $(a-b)(a+b)$

$a^3-b^3 =$ $(a-b)(a^2+ab+b^2)$

$a^3+b^3 =$ $(a+b)(a^2-ab+b^2)$

Example: Factor.

a. $25x^4-30x^2-7$

$(5x^2)^2-6(5x^2)-7$

Let $y=5x^2$. $-7\cdot 1=-7$

$-7+1=-6$

y^2-6y-7

	y	-7
y	y^2	$-7y$
1	$1y$	-7

$(y-7)(y+1)$

$(5x^2-7)(5x^2+1)$

b. $x^3+2x^2-9x-18$

$x^2(x+2)-9(x+2)$

$(x^2-9)(x+2)$

$\boxed{(x-3)(x+3)(x+2)}$

c. $27x^3-64$

$(3x)^3-(4)^3$

$\boxed{(3x-4)(9x^2+12x+16)}$

$(3x-4)((3x)^2+(3x)(4)+4^2)$

Notations

(dividend)÷(divisor)=quotient

OR

dividend=(divisor)(quotient)

If the factor is 2x-1, the root is $\frac{1}{2}$.

If the root is 5, the factor is $x-5$.

A factor is a factor of a polynomial if … the remainder is 0.

Remainder or Factor Theorem

Example: Determine whether $(x+3)$ is a factor of $f(x)=2x^3-x^2-3x+9$ and explain.

$x+3=0 \quad x=-3$

$f(-3)=2(-3)^3-(-3)^2-3(-3)+9$

$=2(-27)-9+9+9$

$=-54+9$

$=-45$

x+3 is not a factor because the remainder is -45.

Rational Root Theorem

Possible: the ones that COULD

Actual: the ones that DO

Example: Find all of the POSSIBLE rational roots of the equation.

$3x^4-5x^3+7x-15=0$

$p:(-15): \pm1, \pm3, \pm5, \pm15$

$q:(3): \pm1, \pm3$

$\frac{p}{q}: \pm\frac{1}{1}, \pm\frac{3}{1}, \pm\frac{5}{1}, \pm\frac{15}{1}, \pm\frac{1}{3}, \pm\frac{3}{3}, \pm\frac{5}{3}, \pm\frac{15}{3}$

$\pm1, \pm3, \pm5, \pm15, \pm\frac{1}{3}, \pm\frac{5}{3}$

How can you find out which of the POSSIBLE roots are ACTUAL roots?

Synthetic Division

Long Division

Remainder Theorem

Polynomial Equation Word Problem

Amazon is trying to decide on the new size for their delivery boxes. They need the boxes to have a volume of 135 in^3. The width of this new box needs to be 3 inches longer than twice the length and the height needs to be 2 inches shorter than the length. Find the dimensions of the box.

$V = 135 in^3 \quad W = 2l+3 \quad h = l-2$

$135 = l(2l+3)(l-2)$

$135 = (2l^2+3l)(l-2)$

$135 = 2l^3 - 4l^2 + 3l^2 - 6l$

$135 = 2l^3 - l^2 - 6l$

$-135 \qquad\qquad -135$

$0 = 2l^3 - l^2 - 6l - 135$

$p: (-135): \pm 1, \pm 3, \pm 5, \pm 9, \pm 15, \pm 27, \pm 45, \pm 135$

$q: (2): \pm 1, \pm 2$

$\frac{p}{q}: \pm\frac{1}{1}, \pm\frac{3}{1}, \pm\frac{5}{1}, \pm\frac{9}{1}, \pm\frac{15}{1}, \pm\frac{27}{1}, \pm\frac{45}{1}, \pm\frac{135}{1}, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{5}{2}, \pm\frac{9}{2}, \pm\frac{15}{2}, \pm\frac{27}{2}, \pm\frac{45}{2}, \pm\frac{135}{2}$

$$\begin{array}{r|rrrr} 1 & 2 & -1 & -6 & -135 \\ & & 2 & 1 & -5 \\ \hline & 2 & 1 & -5 & -140 \end{array}$$

$$\begin{array}{r|rrrr} -1 & 2 & -1 & -6 & -135 \\ & & -2 & 3 & 3 \\ \hline & 2 & -3 & -3 & -132 \end{array}$$

$$\begin{array}{r|rrrr} 2 & 2 & -1 & -6 & -135 \\ & & 4 & 6 & 0 \\ \hline & 2 & 3 & 0 & -135 \end{array}$$

$$\begin{array}{r|rrrr} \frac{9}{2} & 2 & -1 & -6 & -135 \\ & & 9 & 36 & 135 \\ \hline & 2 & 8 & 30 & 0 \\ & \frac{2}{2} & \frac{8}{2} & \frac{30}{2} & \end{array}$$

$(2x-9)(x^2+4x+30)$

$l = \frac{9}{2}$ in OR 4.5 in

$w = 2\left(\frac{9}{2}\right)+3 = 9+3 = 12$ in

$h = \frac{9}{2} - 2 = \frac{9}{2} - \frac{4}{2} = \frac{5}{2}$ in OR 2.5 in

The box should be 4.5 in by 12 in by 2.5 in.