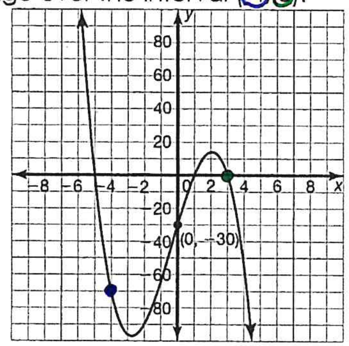
Average Rate of Change

Formula: $\frac{f(b)-f(a)}{b-a}$

Example: Find the average rate of change over the interval (-4, 3).



$$\frac{-70-0}{-4-3} = \frac{-70}{-7} = +10$$

Polynomial Long Division

Example: Write the dividend as the product of the quotient and the divisor.

$(2x^3+3x^2+7x+5) \div (2x+1)$

$$\begin{array}{r} x^2 + x + 3 + \frac{2}{2x+1} \\ 2x+1 \overline{)\,2x^3+3x^2+7x+5} \\ -(2x^3+x^2) \\ 2x^2+7x+5 \\ -(2x^2+x) \\ 6x+5 \\ -(6x+3) \\ 2 \end{array}$$

$2x^3+3x^2+7x+5=$

$(2x+1)(x^2+x+3+\frac{2}{2x+1})$

Synthetic Division

Example: Write the dividend as the product of the quotient and the divisor.

$(x^3-9x^2+8x+60) \div (x-5)$

$$\begin{array}{c|cccc} 5 & 1 & -9 & 8 & 60 \\ & & 5 & -20 & -60 \\ \hline & 1 & -4 & -12 & \boxed{0} \end{array}$$

$x^3-9x^2+8x+60=$

$(x-5)(x^2-4x-12)$

$(4x^3-5x-12) \div (2x+1)$

$$\begin{array}{c|cccc} -\frac{1}{2} & 4 & 0 & -5 & -12 \\ & & -2 & 1 & 2 \\ \hline & \frac{4}{2} & \frac{-2}{2} & \frac{-4}{2} & -10 \end{array}$$

$4x^3-5x-12=(2x+1)(2x^2-x-2-\frac{10}{2x+1})$

Factoring

$a^2-b^2=$ $(a-b)(a+b)$

$a^3-b^3=$ $(a-b)(a^2+ab+b^2)$

$a^3+b^3=$ $(a+b)(a^2-ab+b^2)$

Example: Facor.

a. $25x^4-30x^2-7$

$(5x^2)^2-6(5x^2)-7$

Let $u=5x^2$.

u^2-6u-7

$a \cdot c = 1 \cdot (-7) = -7$

$\frac{-7}{u} \cdot \frac{1}{-7} = -7 \quad \frac{-7}{} + \frac{1}{} = -6$

u	u^2	$-7u$
1	$1u$	-7

$(u-7)(u+1)$

$(5x^2-7)(5x^2+1)$

Not factorable because $5x^2$ and 7 are not perfect square

b. $x^3+2x^2-7x-14$

$x^2(x+2)-7(x+2)$

$(x^2-9)(x+2)$

$(x)^2-(3)^2$

$(x-3)(x+3)(x+2)$

c. $27x^3-64$

$(3x)^3-4^3$

$(3x-4)((3x)^2+(3x)(4)+4^2)$

$(3x-4)(9x^2+12x+16)$

Notations

(dividend) ÷ (divisor) = quotient

OR

dividend = (divisor)(quotient)

If the factor is 2x-1, the root is $\frac{1}{2}$.

If the root is 5, the factor is $x-5$.

Always opposite

A factor is a factor of a polynomial if ... the remainder is 0

Remainder or Factor Theorem

Example: Determine whether (x+3) is a factor of $f(x)=2x^3-x^2-3x+9$ and explain.

If $x+3$ is a factor, $f(-3)=0$.

$f(-3)=2(-3)^3-(-3)^2-3(-3)+9$

$=2(-27)-9+9+9$

$=-54-9+9+9$

$=-45$

Since the remainder is -45, x+3 is not a factor.

Rational Root Theorem

Possible: MAYBE

Actual: FOR REAL; SURE

Example: Find all of the POSSIBLE rational roots of the equation.

$3x^4-5x^3+7x-15=0$

p: -15; ±1, ±3, ±5, ±15

q: 3; ±1, ±3

$\frac{p}{q}: \pm\frac{1}{1}, \pm\frac{3}{1}, \pm\frac{5}{1}, \pm\frac{15}{1}, \pm\frac{1}{3}, \pm\frac{5}{3}$

How can you find out which of the POSSIBLE roots are ACTUAL roots?

You can use long division, synthetic division, or the remainder theorem.